Equations Quadratic in Form

An equation is ***quadratic in form*** if it can be written as $a(\ )^2 + b(\ ) + c = 0,\ a \neq 0,$ where the same expression is in both sets of parentheses.

Solve Equations Quadratic in Form

1. Check to see if the equation is quadratic in form. If it is quadratic in form, the equation will have two variable expressions, and one will be the square of the other.
2. If the equation is quadratic in form, use "*u*-substitution." Let *u* = variable expression of the 2nd term.
3. Solve the equation for "*u*" using quadratic methods (factoring, square root principle, completing the square, quadratic formula).
4. Plug the substitution back in for *u* and solve for the original variable.
5. Check for extraneous solutions.

Examples:

a) $x^4 - 10x^2 + 25 = 0$ $\quad u = x^2$

$u^2 - 10u + 25 = 0$

$(u-5)^2 = 0$

$u = 5$

$x^2 = 5$

$\boxed{x = \pm\sqrt{5}}$

b) $2c^6 - 7c^3 + 3 = 0$ $\quad u = c^3$

Factors of $2(3)=6$	Add to -7
$-6, -1$	-7

$\underline{2}u^2 - 7u + \underline{3} = 0$

$(2u^2 - 6u) + (-u + 3) = 0$

$2u(u-3) - 1(u-3) = 0$

$(u-3)(2u-1) = 0$

$u = 3$ or $u = \frac{1}{2}$

$c^3 = 3$ or $c^3 = \frac{1}{2}$

$\boxed{c = \sqrt[3]{3} \text{ or } c = \sqrt[3]{\frac{1}{2}}}$

c) $(2z+5)^2 - (2z+5) - 6 = 0$ $\quad u = 2z+5$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = 3$ or $u = -2$

$2z + 5 = 3$ or $2z + 5 = -2$

$2z = -2$ $\quad 2z = -7$

$\boxed{z = -1 \text{ or } z = -\frac{7}{2}}$

d) $(x^2-1)^2 - (x^2-1) - 2 = 0$ $\quad u = x^2 - 1$

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2$ or $u = -1$

$x^2 - 1 = 2$ or $x^2 - 1 = -1$

$x^2 = 3$ or $x^2 = 0$

$\boxed{x = \pm\sqrt{3} \text{ or } x = 0}$

e) $p - 3\sqrt{p} - 4 = 0$ $\quad u = \sqrt{p}$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$u = 4$ or $u = -1$

$\sqrt{p} = 4$ or $\sqrt{p} = -1$ Impossible. The $\sqrt{\ }$ symbol means the positive root.

$(\sqrt{p})^2 = 4^2$ $\quad (\sqrt{p})^2 = (-1)^2$

$\boxed{p = 16}$ or $\cancel{p = 1}$

check:

$16 - 3\sqrt{16} - 4 \stackrel{?}{=} 0$

$16 - 3(4) - 4 = 0$ ✓

$1 - 3\sqrt{1} - 4 \stackrel{?}{=} 0$

$1 - 3(1) - 4 \neq 0$

f) $x^{1/2} - 3x^{1/4} + 2 = 0$ $\quad u = x^{1/4}$ (or $\sqrt[4]{x}$)

$u^2 - 3u + 2 = 0$

$(u-2)(u-1) = 0$

$u = 2$ or $u = 1$

$x^{1/4} = 2$ or $x^{1/4} = 1$

$(x^{1/4})^4 = 2^4$ or $(x^{1/4})^4 = 1^4$

$\boxed{x = 16 \text{ or } x = 1}$

check:

$16^{1/2} - 3(16)^{1/4} + 2 \stackrel{?}{=} 0$

$4 - 3(2) + 2 = 0$ ✓

$1^{1/2} - 3(1)^{1/4} + 2 \stackrel{?}{=} 0$

$1 - 3(1) + 2 = 0$ ✓

g) $2m^{-2} + m^{-1} = 15$ $\quad u = m^{-1}$ (or $\frac{1}{m}$)

Factors of $2(-15) = -30$	Add to 1
$6, -5$	1

$2u^2 + u = 15$

$2u^2 + u - 15 = 0$

$(2u^2 + 6u) + (-5u - 15) = 0$

$2u(u+3) - 5(u+3) = 0$

$(u+3)(2u-5) = 0$

$u = -3$ or $u = \frac{5}{2}$

$m^{-1} = -3$ or $m^{-1} = \frac{5}{2}$

$\boxed{m = -\frac{1}{3} \text{ or } m = \frac{2}{5}}$

h) $\dfrac{1}{(x-1)^2} + \dfrac{4}{x-1} = 12$

$\left(\frac{1}{x-1}\right)^2 + 4\left(\frac{1}{x-1}\right) - 12 = 0$ $\quad u = \frac{1}{x-1}$

$u^2 + 4u - 12 = 0$

$(u+6)(u-2) = 0$

$u = -6$ or $u = 2$

$\frac{1}{x-1} = -6$ or $\frac{1}{x-1} = 2$

$1 = -6(x-1)$ or $1 = 2(x-1)$

$1 = -6x + 6$ $\quad 1 = 2x - 2$

$-5 = -6x$ $\quad 3 = 2x$

$\boxed{x = \frac{5}{6} \text{ or } x = \frac{3}{2}}$